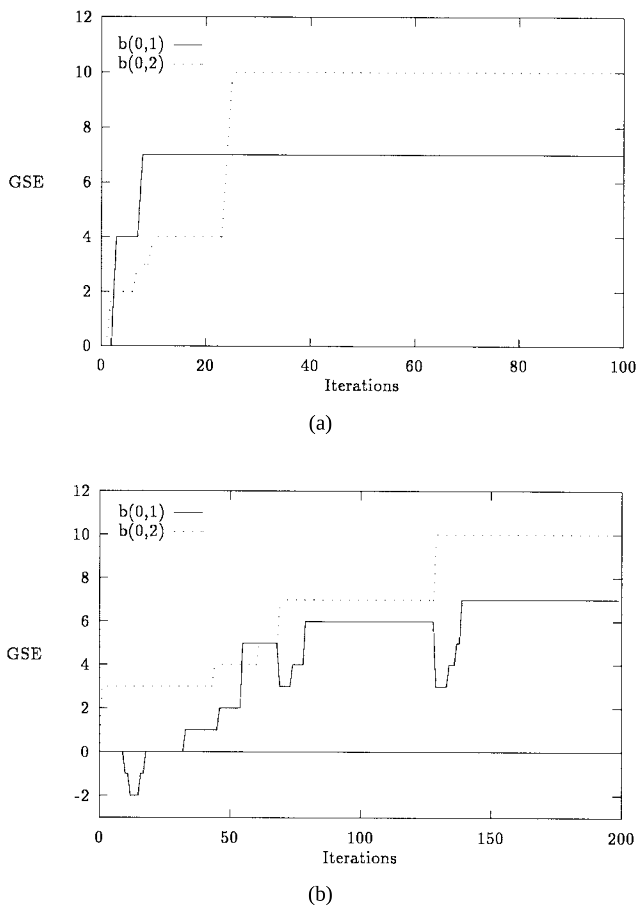

Fig. 10. The convergence curve of linearly independent elements in the basis matrix. (a) Opening. (b) Closing.

V. Conclusions

We presented a method for optimizing the gray-scale function processing morphological filters under the LMS error criterion. To determine the optimal structuring elements of erosion and dilation, an adaptation rule using the estimation of gradient of the MSE was presented. Based on this approach, we proposed an adaptation algorithm for the basis matrix for opening and closing, which was introduced for efficient implementation of opening and closing. The LMS and backpropagation algorithms were utilized for obtaining the optimal basis matrix for opening and closing. Experimental results indicated that adaptation process under the LMS error criterion was converged in a few iterations.

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Simultaneous Multichannel Image Restoration and Estimation of the Regularization Parameters

Moon Gi Kang and Aggelos K. Katsaggelos

***Abstract*— In this correspondence, a constrained least-squares multichannel image restoration approach is proposed, in which no prior knowledge of the noise variance at each channel or the degree of smoothness of the original image is required. The regularization functional for each channel is determined by incorporating both within-channel and cross-channel information. It is shown that the proposed smoothing functional has a global minimizer.**

I. Introduction

Considerable attention has been focused recently on image restoration techniques that are based on a multichannel formulation, since improved results can be obtained by incorporating cross-channel information into the restoration process [1]–[5]. In this correspondence, the following multichannel degradation model is considered:

$$g = Dx + n \tag{1}$$

where g, x, and n represent the observed image, the original image, and the noise, respectively. For an N channel image, they are described by $g^T = [g_1^T, g_2^T, \cdots, g_N^T]$, $x^T = [x_1^T, x_2^T, \cdots, x_N^T]$, and $n^T = [n_1^T, n_2^T, \cdots, n_N^T]$, respectively, where each channel g_i, x_i, and n_i is a vector of dimension M^2, resulting from the ordering of each $M \times M$ channel image. The multichannel degradation matrix D is defined by

$$D = \begin{bmatrix} D_{11} & D_{12} & \cdots & D_{1N} \\ D_{21} & D_{22} & \cdots & \cdot \\ \vdots & \vdots & \cdots & \vdots \\ D_{N1} & D_{N2} & \cdots & D_{NN} \end{bmatrix} \tag{2}$$

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where each D_{ij} is of dimensions $M^2 \times M^2$. The diagonal matrices D_{ii} represent the within-channel blur, and the off-diagonal matrices D_{ij} represent the cross-channel blur.

The multichannel restoration problem is then to obtain an estimate of x given g, D and possibly some knowledge on n. Optimal single-channel restoration filters do not produce optimal results when applied to the solution of (1), even when $D_{ij} = 0$, $i \neq j$, since the between-channel relations are not utilized. Stochastic multichannel linear minimum mean square error filters were considered in [1]–[3], [5]. A serious drawback of these filters is the sensitivity to the required estimates of the between-channel correlations [3].

Regularized least-squares filters that alleviate this difficulty of stochastic multichannel filters were proposed in [4]. A major issue with this approach is the determination of the regularization parameters, which is performed separately. Besides being a computationally expensive step, it also requires prior knowledge of the noise statistics and the quantification of the degree of smoothness of the required solution.

In this correspondence, a constrained least-squares approach is followed in solving (1). We propose an iterative algorithm for the simultaneous estimation of the regularization parameters and the original image, by applying the paradigm we introduced in [6]–[8]. The functional to be minimized is appropriately chosen so that it is convex and, therefore, it has a unique minimizer, as shown in Section II. The properties of the proposed iterative algorithm are described in Section III, and a choice of the regularization functional in Section IV. Experimental results are shown in Section V, and we conclude with Section VI.

II. Problem Formulation

Equation (1) is rewritten as [4]

$$g_i = D_i x + n_i, \qquad i = 1, 2, \cdots, N \tag{3}$$

where

$$D_i = [D_{i1}, D_{i2}, \cdots, D_{iN}]$$

is the ith block row matrix of dimension $M^2 \times NM^2$. We propose to obtain a solution to (3) by minimizing the smoothing functional

$$Z(x) = \sum_i^N Z_i[\alpha_i(x), x] \tag{4}$$

where

$$Z_i[\alpha_i(x), x] = \|g_i - D_i x\|^2 + \alpha_i(x)\|C_i x\|^2. \tag{5}$$

Each C_i in (5) is written as

$$C_i = [C_{i1}, C_{i2}, \cdots C_{iN}], \qquad i = 1, 2, \cdots N \tag{6}$$

where C_{ij} is an $M^2 \times M^2$ matrix representing a highpass filter, which imposes smoothness within each channel with C_{ii}, and across channels with C_{ij}, $i \neq j$. The regularization functional $\alpha_i(x)$ controls the relative contribution of the error term for the ith channel, $\|g_i - D_i x\|^2$, and the stabilizing functional $\|C_i x\|^2$. It is explicitly expressed as a function of the original image. The smoothing functional $Z(x)$ is in general nonlinear, and it has more than one minimizers. In order for it to have a global minimizer, $\alpha_i(x)$ should be chosen in a proper way. We show next the properties $\alpha_i(x)$ should satisfy, extending our results for single-channel images [8]. Toward this end, the following proposition is needed.

Proposition 1: Summation of convex functionals results in a convex functional.

The proof of this proposition can be shown in a straightforward manner. According to it, if each functional $Z_i[\alpha_i(x), x]$ in (5) is convex, the smoothing functional $Z(x)$ in (4) is convex.

The analysis now focuses on the determination of the $\alpha_i(x)$'s so that each $Z_i[\alpha_i(x), x]$ is convex, while the $\alpha_i(x)$'s retain their purpose of controlling the relative contribution of the error term of each channel $\|g_i - D_i x\|^2$, which enforces "faithfulness" to the data, and the stabilizing functional of each channel $\|C_i x\|^2$, which enforces smoothness on the solution.

For this purpose, each $\alpha_i(x)$ should have the following properties:

Property 1: $\alpha_i(x)$ should be a function of the regularized noise power at the ith channel; that is

$$\alpha_i(x) = f\{Z_i[\alpha_i(x), x]\} \tag{7}$$

where $f(\cdot)$ represents a monotonically increasing function. The justification behind this choice of $\alpha_i(x)$ is based on the set theoretic formulation of the restoration problem [9] and is described in [8].

Property 2: The minimizer of the multichannel smoothing functional $\hat{x}$ that satisfies $\nabla_x Z(\hat{x}) = 0$, should also satisfy $\nabla_x[\sum_i^N \alpha_i(\hat{x})] = 0$. This is indeed the case when $f(\cdot)$ is linear, since

$$\begin{aligned}\nabla_x \sum_i^N \alpha_i(\hat{x}) &= \nabla_x \sum_i^N f\{Z_i[\alpha_i(\hat{x}), \hat{x}]\} \\ &= \nabla_x f\{\sum_i^N Z_i[\alpha_i(\hat{x}), \hat{x}]\} \\ &= \frac{df}{dZ}\nabla_x Z(\hat{x})\end{aligned} \tag{8}$$

which is equal to zero when $\nabla_x Z(\hat{x}) = 0$.

Property 3: The $\alpha_i(x)$s should be chosen in such a way that each smoothing functional $Z_i[\alpha_i(x), x]$ is convex. The sufficient condition on $\alpha_i(x)$ for each $Z_i[\alpha_i(x), x]$ to be convex is given by [8]

$$\frac{\partial f(Z_i)}{\partial Z_i} \leq \frac{1}{\|C_i x\|^2}. \tag{9}$$

Due to Proposition 1, the convexity of each $Z_i[\alpha_i(x), x]$ results in the convexity of $Z(x)$. Then a local extremum of it becomes a global extremum [10].

With the choice of the regularization functional that satisfies the three properties described above, the equation for solution is obtained by taking the gradient of the smoothing functional $Z(x)$ and setting it equal to zero (necessary condition for a minimum). That is

$$\begin{aligned}\nabla_x Z(x) = \sum_{i=1}^N \{&[D_i^T D_i + \alpha_i(x) C_i^T C_i]x \\ &- D_i^T g_i + \|C_i x\|^2 \nabla_x \alpha_i(x)\} = 0.\end{aligned} \tag{10}$$

Since $\sum_{i=1}^N \nabla_x \alpha_i(x) = 0$ when $\nabla_x Z(x) = 0$ according to Property 2, (10) becomes

$$\sum_{i=1}^N [D_i^T D_i + \alpha_i(x) C_i^T C_i]x = \sum_{i=1}^N D_i^T g_i. \tag{11}$$

Equation (11) was derived in [4] for $\alpha_i(x) = \lambda_i$, the regularization parameter for each channel, which needs to be defined independently.

III. Iterative Solution

Equation (11) can not be solved for x in a direct way, since it is nonlinear with respect to x. Therefore, a successive approximations iteration is used resulting in

$$x_{k+1} = x_k + \beta\left[\sum_{i=1}^{N} D_i^T g_i - \sum_{i=1}^{N}(D_i^T D_i + \alpha_i(x) C_i^T C_i) x_k\right]. \quad (12)$$

The following proposition provides a sufficient condition for the convergence of iteration (12).

Proposition 2: Consider the equation

$$G(x)x = b \quad (13)$$

where $G(x)$ is a square matrix, $G(x) = G_1(x) + G_2(x) + \cdots + G_N(x)$, $b = b_1 + b_2 + \cdots + b_N$, and each $G_i(x)$ is a positive definite matrix. Consider also the iteration

$$x_{k+1} = x_k + \beta[b - G(x_k)x_k] \quad (14)$$

for solving (13). If

$$\beta \cdot \sigma_{\max}[J_{G_i(x)x}] < \frac{2}{N}, \qquad \forall i \quad (15)$$

where $J_{G_i(x)x}$ is the Jacobian matrix of the vector $G_i(x)x, \sigma_{\max}(\cdot)$ its maximum singular value and β the relaxation parameter, then (14) converges to a solution of (13).

By applying Proposition 2 to iteration (12) the sufficient condition for convergence of the multichannel iteration becomes

$$\begin{aligned}\sigma_{\max}&[D_i^T D_i + \alpha_i(x) C_i^T C_i] \\ &\le \sigma_{\max}(D_i^T D_i) + \alpha_i(x)\sigma_{\max}(C_i^T C_i) \\ &< \frac{2}{\beta N}, \qquad \text{for } i = 1, 2, \cdots, N\end{aligned} \quad (16)$$

and therefore

$$\alpha_i(x) < \frac{2 - \beta N \sigma_{\max}(D_i^T D_i)}{\beta N \sigma_{\max}(C_i^T C_i)}, \qquad \text{for } i = 1, 2, \cdots, N. \quad (17)$$

In summary, if (17) is satisfied, iteration (12) converges to a (local) minimum of $Z(x)$. In addition, if (9) is satisfied, $Z(x)$ is convex, and therefore iteration (12) converges to its unique minimizer.

IV. Choice of the Multichannel Regularization Functional

A regularization functional that satisfies the three properties described in the previous section is determined by a linear function $f(\cdot)$, that is

$$\alpha_i(x) = f\{Z_i[\alpha_i(x), x]\} = \gamma Z_i[\alpha_i(x), x] \quad (18)$$

where γ is determined by the sufficient conditions for convergence and convexity. From (18), $\alpha_i(x)$ is written as

$$\alpha_i(x) = \frac{\|g_i - D_i x\|^2}{\frac{1}{\gamma} - \|C_i x\|^2}, \qquad \text{for } i = 1, 2, \cdots, N \quad (19)$$

with $1/\gamma \ge \|C_i x\|^2$ resulting from the condition for convexity. From the sufficient condition for convergence (17), we have

$$\frac{\|g_i - D_i x\|^2}{\frac{1}{\gamma} - \|C_i x\|^2} < \frac{2 - \beta N \sigma_{\max}(D_i^T D_i)}{\beta N \sigma_{\max}(C_i^T C_i)}, \qquad \text{for } i = 1, 2, \cdots, N \quad (20)$$

and therefore

$$\frac{1}{\gamma} > \frac{\beta N \|g_i - D_i x\|^2 \sigma_{\max}(C_i^T C_i)}{2 - \beta N \sigma_{\max}(D_i^T D_i)} + \|C_i x\|^2. \quad (21)$$

TABLE I
VALUES OF THE REGULARIZATION PARAMETERS AT CONVERGENCE WITH VARIOUS INITIAL CONDITIONS

x_0	R channel	G channel	B channel
$D^T g$	0.00895	0.0101	0.00520
$Q^T g$	0.00893	0.0101	0.00519
$D^T Q g$	0.00894	0.0100	0.00520
Mixed	0.00894	0.0101	0.00520

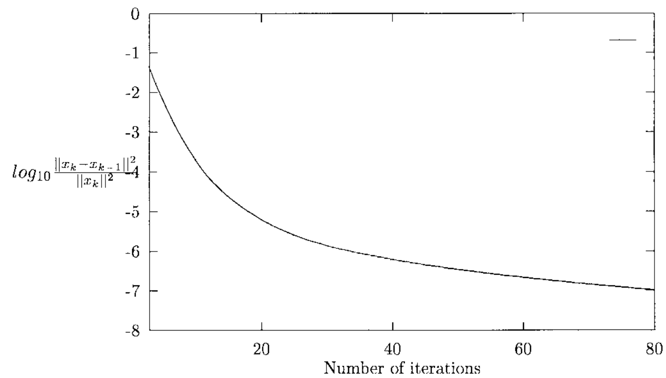

Fig. 1. Values of $\log_{10}(\|x_k - x_{k-1}\|^2/\|x_k\|^2)$.

Since $\sigma_{\max}(D_i^T D_i) = \sigma_{\max}(C_i^T C_i) = 1$, for overall channel energy preservation, if we choose $\beta = 1/N$, condition (21) becomes

$$\frac{1}{\gamma} > \|g_i - D_i x\|^2 + \|C_i x\|^2. \quad (22)$$

Now $\|g\|^2 \ge \|g_i - D_i x\|^2$, since all elements of $D_i x$ are positive, and $\|C_i x\|^2 \le \|g\|^2$, since x is assumed to have more energy at low than high frequencies. Therefore, the choice $1/\gamma = 2\|g\|^2$ satisfies the condition for convergence (22), and also provides a positive $\alpha_i(x)$ according to (19).

V. Experimental Results

The performance of the proposed multichannel restoration algorithm has been investigated with artificially blurred three-channel color images. Some of the restoration results are presented in this section, for the 256 × 256 pixels portrait color image, Lena. The point spread function (PSF) of the blurring system within and across channels is Gaussian with variance 9. For the within-channel PSF $d_{ii}(m, n)$, it holds that $\sum_{m,n} d_{ii}(m, n) = 0.8$, and for the cross-channel blur PSF $d_{ij}(m, n)$ it holds that $\sum_{m,n} d_{ij}(m, n) = 0.1$. The three-dimensional (3-D) Laplacian was used as the multichannel stabilizing operator C, the criterion $\|x_{k+1} - x_k\|^2/\|x_k\|^2 \le 10^{-7}$ for terminating the iteration, and β was set equal to 1/3, according to the convergence analysis presented in the previous section.

In order to show the global optimality of the proposed multichannel restoration algorithm, it was run with various initial values $x_0^T = [x_{0R}^T, x_{0G}^T, x_{0B}^T]$. A lowpass filtered version of the multichannel observed image $D^T g$, a highpass filtered version of the multichannel observed image $C^T g$, a bandpass filtered version of the multichannel observed image $CD^T g$ and a three-channel image consisting of a lowpass filtered red channel, a highpass filtered green channel, and a bandpass filtered blue channel, were used as x_0. The values of the regularization functional at convergence in all these cases are shown in Table I. This shows the global optimality of the proposed algorithm. The proposed algorithm also provides as a byproduct an estimate of the variance of the noise at each channel $\|y - Dx_k\|^2/M^2$. The estimated value for 30 dB SNR is slightly bigger than the

Fig. 2. Noisy-blurred image for a cross-channel Gaussian blue; R channel.

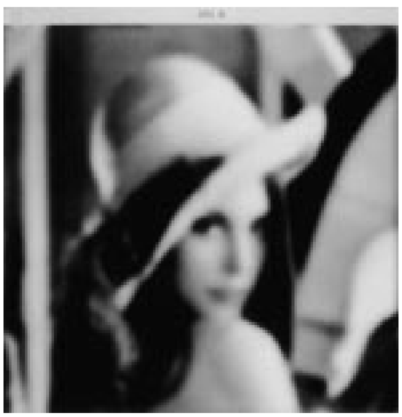

Fig. 3. Noisy-blurred image for a cross-channel Gaussian blue; G channel.

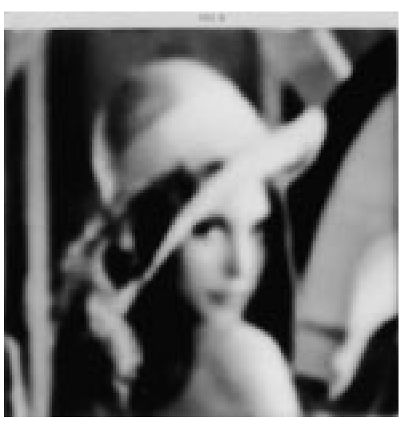

Fig. 4. Noisy-blurred image for a cross-channel Gaussian blur; B channel.

Fig. 5. Restored image by the proposed algorithm; R channel.

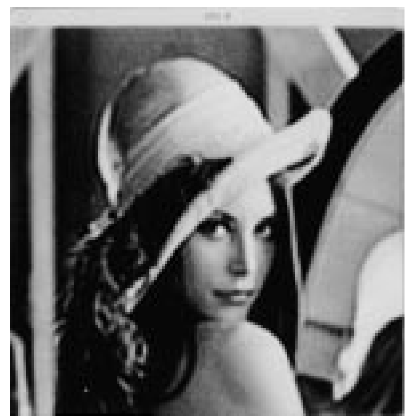

Fig. 6. Restored image by the proposed algorithm; G channel.

Fig. 7. Restored image by the proposed algorithm; B channel.

actual one (estimated values for red-green-blue (RGB) channels: 2.27, 2.67, 1.33; corresponding actual values: 1.47, 1.62, 0.754). The convergence and the rate of convergence are demonstrated by showing the normalized step difference, i.e., $\|x_k - x_{k-1}\|^2/\|x_k\|^2$ in Fig. 1 . The three noisy blurred channels are shown in Figs. 2–4, and the corresponding restored channels are shown in Figs. 5–7.

VI. Conclusion

In our recent work [8], we have introduced a paradigm according to which the single image restoration problem is formulated as a nonlinear minimization problem, which is guaranteed to have a unique minimum. The required parameters for the solution are estimated iteratively based on the partially restored image. In this correspondence, we extend the application of this paradigm to the restoration of multichannel images. The proposed algorithm does not require any prior knowledge about the noise or the original image, other than knowledge of the degradation matrix D. Then after the C_i's are determined, the algorithm operates on the available noisy-blurred image and results in an accurate restored image (based on the iterative determination of the regularization parameter), as well as an accurate estimate of the variance of the additive noise at each channel. The analysis holds true, and the algorithm is applicable to any type of degradation D, which implies spatially variant blurs.

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