VIDEO RETRIEVAL USING SPARSE BAYESIAN RECONSTRUCTION

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ABSTRACT

Every day, a huge amount of video data is generated for different purposes and applications. Fast and accurate algorithms for efficient video search and retrieval are therefore essential. The interesting properties of sparse representation and the new sampling theory named Compressive Sensing (CS) constitute the core of the new approach to video representation and retrieval we are presenting in this paper. Once the representation (where sparsity is expected) has been chosen and the observations have been taken, the proposed approach utilizes Bayesian modeling and inference to tackle the retrieval problem. In order to speed up the inference process the use of Principal Components Analysis (PCA) to provide an alternative representation of the frames is analyzed. Experimental results validate the proposed approach whose robustness against noise is also examined.

***Index Terms*—** Video retrieval, compressive sensing, Bayesian modeling, Bayesian inference

1. INTRODUCTION

A large amount of video data is generated every day. Searching through huge video databases is an important problem in many applications. For instance, individuals may want to search for video content they are interested in from YouTube videos, media companies may want to locate video content that violates their copyright protection (fingerprint) and, security systems may want to detect suspicious events among surveillance videos. Fast and accurate algorithms in all these cases are needed for efficient video retrieval.

Due to the different types of query applications (such as query by example, query by video clip, query by semantics, etc), various image/video features are being employed by the different algorithms. For example, the color histogram of video frames is used in [1] , both color and motion features are used in [2, 3, 4, 5, 6, 7], visual features and semantic labels are used in [8], and time interval statistics are used in [9]. A survey of this topic can be found in [10]. In [11], the authors compared the use of local and global features.

With the former robust results are obtained with high computational cost, while with the latter computational efficiency is gained at the expense of reduced performance.

Some algorithms also use indexing or hashing to improve search efficiency. For example, in [8] geometric hashing is used to build database indices, while in [4, 5, 7, 9] indexing tree structures are used. In [12], a kd-tree based space partitioning indexing scheme is applied to the video trajectory representations by using scaling and PCA. In [13] several random projections are used to project scaled videos on different search spaces, and then kd-trees on each space are used.

As described in [14] sparsity has emerged in the last decade as one of the important concepts in a wide range of signal processing applications [15, 16]. This interest has been even more elevated by the compressive sensing (CS) theory [17, 18, 19]. Compressive sensing is a new paradigm for signal acquisition where a signal is recovered from a low number of measurement without satisfying the Nyquist rate. CS is based on two main principles. First, the signal of interest can be represented with a sparse set of coefficients in a basis

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($\mathbf{B}_S$).The second important property is the incoherence between this representation basis and the measurement basis. It is shown by a large body of work that CS can be applied with great success to many application dramatically reducing the number of measurements needed for signal reconstruction.

Originally, most of the works in the CS and sparse representation literature focused on accurate representation and recovery of a signal in a given dictionary or basis. More recent works, however, exploit the discriminative properties of sparse recovery for classification (see, for instance, [20, 21]). The general principle behind sparse recovery for classification is that the test signals can be represented as linear combinations of the samples in the dictionary. Generally, this linear combination will include only a few coefficients, thus choosing the most relevant samples in the dictionary.

In this paper, we exploit the same discriminative nature of sparse representation for the video retrieval problem. Specifically, our goal is to find the sparsest representation of an input query video clip from the samples of a video database. We first construct the video database that is invariant to the starting video frame, and then formulate the video retrieval problem as sparse reconstruction. We employ a Bayesian compressive sensing algorithm to find the sparse representations of query videos within this database, and apply the classification procedure on the recovered sparse coefficients. Empirical results demonstrate the high retrieval performance of the proposed method compared to some existing algorithms.

The paper is organized as follows. In section 2 we explain how retrieving a video clip can be formulated as finding sparse representation in a convenient domain. In section 3 we formulate the video retrieval problem using the Bayesian framework, describe the inference procedure and explain the classification method for deciding whether a query video is in the database. In section 4 we discuss the feature extraction procedure. In section 5 we analyze the performance of the proposed system and determine its robustness in comparison with other systems.

2. SPARSE REPRESENTATION OF VIDEO CLIPS

In this section, we build a sparse representation for each video clip in the database in order to retrieve a clip of interest from a database using sparse representation principles.

The video database can be represented as a matrix by concatenating the existing video clips as

$$\mathbf{A} = [\mathbf{a}_{1,1}, \mathbf{a}_{1,2}, \dots, \mathbf{a}_{1,N_1}, \dots, \mathbf{a}_{K,1}, \dots, \mathbf{a}_{K,N_K}] \tag{1}$$

where $\mathbf{a}_{i,j},\ i = 1, \dots, K, j = 1, \dots, N_i$ represents the $j-$th frame in the $i-$th video. Each $\mathbf{a}_{i,j}$ is assumed to be a column vector of size M where $M = VH$ with V and H the vertical and horizontal dimensions of each frame. For notational convenience, Eq. (1) is rewritten as

$$\mathbf{A} = [\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_N] \tag{2}$$

where $N = \sum_{i=1}^{K} N_i$. Let $\mathbf{y}$ be a video-clip in the database written in the vector form as

$$\mathbf{y} = \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \vdots \\ \mathbf{y}_S \end{bmatrix} \tag{3}$$

where each $\mathbf{y}_i$ represents the $i-$th frame, and S is the length of the video clip. Next, we build the following matrix for a query of length S

$$\tilde{\mathbf{A}} = \begin{pmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 & \dots & \mathbf{a}_{N-(S-1)} \\ \mathbf{a}_2 & \mathbf{a}_3 & \mathbf{a}_4 & \dots & \mathbf{a}_{N-S} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{a}_S & \mathbf{a}_{S+1} & \mathbf{a}_{S+2} & \dots & \mathbf{a}_N \end{pmatrix} \tag{4}$$

by shifting the columns of $\mathbf{A}$ in (1). Using this matrix, it can be observed that $\mathbf{y}$ admits a sparse representation as

$$\mathbf{y} = \tilde{\mathbf{A}}\mathbf{x}_0 \tag{5}$$

where $\mathbf{x}_0 = (0, \dots, 0, 1, 0, \dots, 0)^t$ is a sparse vector with all coefficients equal to zero except for the entry corresponding to the location of the video clip $\mathbf{y}$ in the database, which is equal to 1. Hence, the position of $\mathbf{y}$ in the database is determined by $\mathbf{x}_0$.

For a given clip $\mathbf{y}$, solving for the corresponding $\mathbf{x}_0$ is an ill-posed problem as the system in (5) is highly underdetermined which leads to non-uniqueness of the solutions. However, as our goal is to find the sparsest solution, i.e., finding the solution with most components equal to zero, this motivates following [21] to seek for the solution of

$$\hat{\mathbf{x}}_0 = \arg\min_{\mathbf{x}} \parallel \mathbf{x} \parallel_0 \quad \text{subject to} \quad \tilde{\mathbf{A}}\mathbf{x} = \mathbf{y} \tag{6}$$

where $\parallel \mathbf{x} \parallel_0$ is the l_0-quasinorm (the number of non-zero coefficients). However, as is well known, the solution of this optimization problem is NP-hard. Furthermore, there are other issues like noise, different image sizes or even occlusions that make us resort to the CS formulation of the problem.

The noisy CS acquisition system can be modeled as

$$\mathbf{y} = \tilde{\mathbf{A}}\mathbf{x} + \mathbf{n}, \tag{7}$$

where $\mathbf{n}$ is the $(SM) \times 1$ independent, Gaussian, zero-mean noise vector with variance equal to β^{-1}. The problem (6) can then be relaxed using the l_1-norm formulation as

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}} \{\|\mathbf{y} - \tilde{\mathbf{A}}\mathbf{x}\|_2^2 + \tau\|\mathbf{x}\|_1\}, \tag{8}$$

where $\|\cdot\|_1$ denotes the l_1-norm. Solving (8) is much easier than (6) and has attracted much interest in the CS community.

3. VIDEO RETRIEVAL BASED ON BAYESIAN COMPRESSIVE SENSING

A number of methods have been proposed to solve the sparse optimization problem in Eq. (8), (see [14] and references therein, see also [22]). In this paper, we formulate the problem using the Bayesian framework following [23] which will also allow us to automatically estimate the regularization parameters, (see [23, 14] for references to parameter estimation). We provide here a brief review of solving (8) using a Bayesian approach.

In Bayesian modeling, all unknowns are treated as stochastic quantities with assigned probability distributions. The joint probability distribution of all quantities is given by

$$\mathrm{p}(\mathbf{x}, \boldsymbol{\gamma}, \beta, \mathbf{y}) = \mathrm{p}(\mathbf{y}|\mathbf{x}, \beta)\, \mathrm{p}(\mathbf{x}|\boldsymbol{\gamma})\, \mathrm{p}(\boldsymbol{\gamma})\, \mathrm{p}(\beta). \tag{9}$$

The observation noise is independent and Gaussian with zero mean and variance equal to β^{-1}, that is, with (7),

$$\mathrm{p}(\mathbf{y}|\mathbf{x}, \beta) = \mathcal{N}(\mathbf{y}|\tilde{\mathbf{A}}\mathbf{x}, \beta^{-1}). \tag{10}$$

It is shown in [23] that the l_1 regularization formulation in (8) is equivalent to using a hierarchical Laplace prior on the coefficients of $\mathbf{x}$, that is,

$$\mathrm{p}(\mathbf{x}|\boldsymbol{\gamma}) = \prod_{i=1}^{N} \mathcal{N}(x_i|0, \gamma_i), \tag{11}$$

$$\mathrm{p}(\gamma_i|\lambda) = \frac{\lambda}{2} \exp\left(-\frac{\lambda\gamma_i}{2}\right), \ \gamma_i \geq 0, \ \lambda \geq 0, \tag{12}$$

where $\boldsymbol{\gamma} = (\gamma_1, \gamma_2, \ldots, \gamma_N)$. Using this specification, the signal distribution $\mathrm{p}(\mathbf{x}|\mathbf{y}, \lambda, \beta)$ is estimated as a multivariate Gaussian distribution $\mathcal{N}(\mathbf{x}|\mu, \Sigma)$ with parameters

$$\Sigma = \left[\beta\tilde{\mathbf{A}}^t\tilde{\mathbf{A}} + \Lambda\right]^{-1}, \tag{13}$$

$$\mu = \Sigma\, \beta\tilde{\mathbf{A}}^t\mathbf{y}, \tag{14}$$

with $\Lambda = \mathrm{diag}(1/\gamma_i)$. The hyperparameters $\boldsymbol{\gamma}$ are then estimated by forming the likelihood function

$$\mathcal{L} = -\frac{1}{2}\log|\mathbf{C}| - \frac{1}{2}\mathbf{y}^t\mathbf{C}^{-1}\mathbf{y} + N\log\frac{\lambda}{2} - \frac{\lambda}{2}\sum_i \gamma_i\,, \tag{15}$$

with $\mathbf{C} = \left(\beta^{-1}\mathbf{I} + \tilde{\mathbf{A}}\Lambda^{-1}\tilde{\mathbf{A}}^t\right)$, and maximizing it with respect to each γ_i and λ in an alternating fashion. This procedure results in the updates

$$\gamma_i = -\frac{1}{2\lambda} + \sqrt{\frac{1}{4\lambda^2} + \frac{<x_i^2>}{\lambda}}\,, \tag{16}$$

$$\lambda = \frac{N-1}{\sum_i \gamma_i/2}, \tag{17}$$

where $<x_i^2> = x_i^2 + \Sigma_{ii}$. In summary, at each iteration of the algorithm, given an estimate of $\boldsymbol{\gamma}$ and λ, the estimate of the distribution of $\mathbf{x}$ is calculated using (13) and (14), followed by the estimation of the variances γ_i from (16) and the hyperparameter λ from (17). In addition, [23] proposed a greedy approach that finds the solutions much more efficiently without the need of solving the large linear system in (14). In our work, we use this greedy approach to find the solution of (8).

3.1. Classification Procedure

We finally proceed to decide whether the query video-clip is in the database. If the query video-clip is in database, then its sparse representation will only have one non-zero component, and equal to 1 in the position of the first frame of the query video. Let $\hat{\mathbf{x}}$ be the vector μ at convergence of the Bayesian algorithm, and $m = \max_i \hat{x}_i$. We then define the vector $\mathbf{x}^{comp}$ with components

$$x_i^{comp} = \begin{cases} 1 & \text{if } \hat{x}_i = m \\ 0 & \text{otherwise} \end{cases} \quad i = 1, 2, \ldots, N. \tag{18}$$

We fix a threshold δ and decide that the query video is in the database if, and only if, $\mathbf{x}^{comp}$ only has one non-zero component and

$$\|\hat{\mathbf{x}} - \mathbf{x}^{comp}\|_1 \ \leq \ \delta \tag{19}$$

4. FEATURE EXTRACTION

In order to perform an efficient search, and due to the size of frames, feature extraction is needed. We first assume that the frames in the database have been downsampled to a reasonable size (11×8 in our experiments). We then use a linear feature transformation. The projection from the image space to the feature space can be represented by a matrix $\mathbf{D} \in \mathbb{R}^{T\times M}$ with $T \ll M$ which when applied to $\mathbf{A}$ produces

$$\mathbf{D}_{T\times M}\mathbf{A}_{M\times N} = \dot{\mathbf{A}}_{T\times N} \tag{20}$$

Then we can use the proposed retrieval procedure on $\dot{\mathbf{A}}$, which leads to a faster search. In this work we consider $\mathbf{D}$ to be the matrix associated to PCA, see [12]. Notice that we could have also used a matrix of random projections $\Phi_{T\times M}$.

The PCA transformation retains much of the information in only a reduced set of principal components. The number of preserved dimensions, T, determines the energy loss during the PCA transformation. The energy represented by each PCA coefficient obtained from the test database used in the experiments, which consists of 567146 frames, is shown in Figure 1. Notice that for T=4 70 % of the energy is preserved. Furthermore if the CS theorical conditions are met by $\tilde{\mathbf{A}}$, see [21], then $\hat{\mathbf{x}}$ can be recovered by l1-minimization with overwhelming probability if $ST > 2log(567146/ST)$. In other words, around $ST \approx 10$ would suffice to recover the only non-zero component. As we will see in the experimental section, when $S = 3$ and $T = 4$ the proposed system retrieves all the relevant clips in the database in the noiseless case.

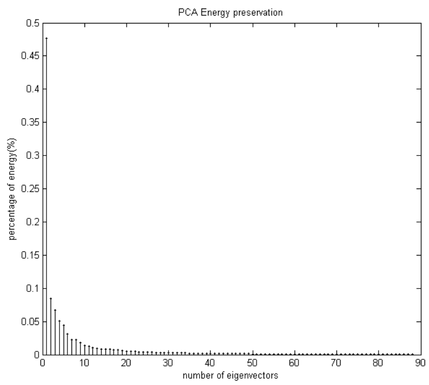


Fig. 1. Energy of the PCA components

No. of frames	Frames in database	Frames not in database
3	1.45 s	2.90 s
7	2.62 s	4.44 s
15	5.72 s	8.07 s

Table 1. CPU time used to find a query.

5. SIMULATION RESULTS

In our experiments we used the 2004 NIST TRECVID shot boundary test set. This data set has approximately 6 hour of video in 12 videos (each of about 30 mins long). We split it in two data sets. The positive video repository (or database) consists of 11 videos and the other video forms the negative data set.

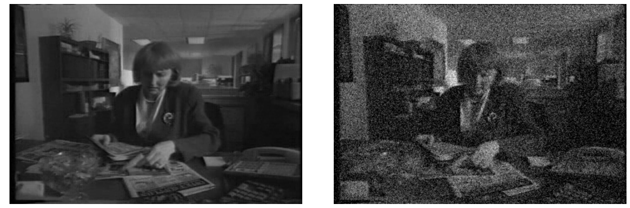
The frames are downsampled with a scaling factor of 32 to produce 11x8 video icons. Then the frames are projected using PCA transformation with $T = 4$. In our test, we select randomly 250 positives and 250 negatives query videos. The query clip lengths are $S = 3, 7$ and 15 frames. All experiments were performed in an Intel Core 2 Duo 2GHz notebook with 2 GB of RAM. The mean times the Bayesian algorithm took to find the sparse representation of a video query is reported in Table 1.

5.1. Noise free test cases

For noise-free test cases our system retrieved all positive cases and rejected all negative one. The results are exactly the same as the ones reported in [12] and [13].

5.2. Noisy test cases

In real world applications video clips can be corrupted by coding and communication losses, as well as, image formation variations. To simulate coding losses in the query clips, we added Gaussian noise to the query clips at PSNR levels of 20, 25, 30, and 35 dB. Figure 2 shows one original image in the database and its 20 dB noisy observation. Due to their sizes, the corresponding 11x8 icon frames are not shown.



(a) (b)

Fig. 2. (a) Original frame in the database, (b) its noisy observation

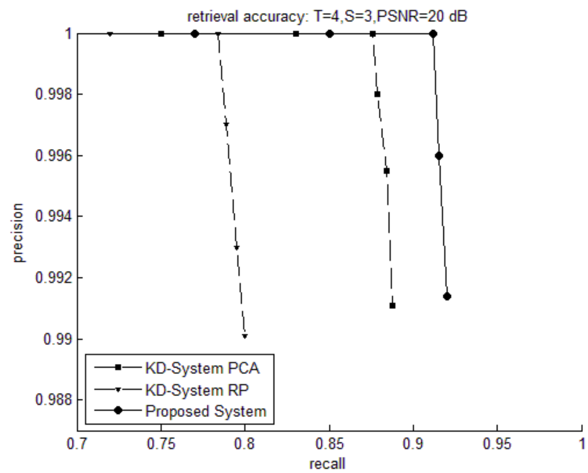


Fig. 3. Precission-Recall curves for three video-retrieval systems: KD-System PCA [12], KD-System RP [13], and Proposed Method.

In order to compare our system with certain state-of-art algorithms, we calculated the precision-recall curves, for the same set of query clips. The precision-recall curve [24] is a typical way of characterizing retrieval performance. For a given threshold, let us assume that a is the number of relevant (present in the database) clips retrieved, b the number of relevant clips not retrieved, and c the number of non relevant clips retrieved, then the precision and recall values are defined by $precision = a/(a+c)$ and $recall = a/(a+b)$, respectively. By changing the threshold value we obtain, for a given method, its precision-recall curve. Notice that as the threshold δ in Eq. (19) decreases the recall value is expected to decrease while the precision value is expected to increase.

In Fig. 3 three systems are compared for the case $T = 4$, $S = 3$, and $PSNR = 20\ dB$. We can see that the method proposed in [13] performs worse than the one in [12]. Furthermore, the proposed sparse Bayesian retrieval method performs better than the method in [12].

In Fig. 4 the same systems compared for the case $T = 4$,

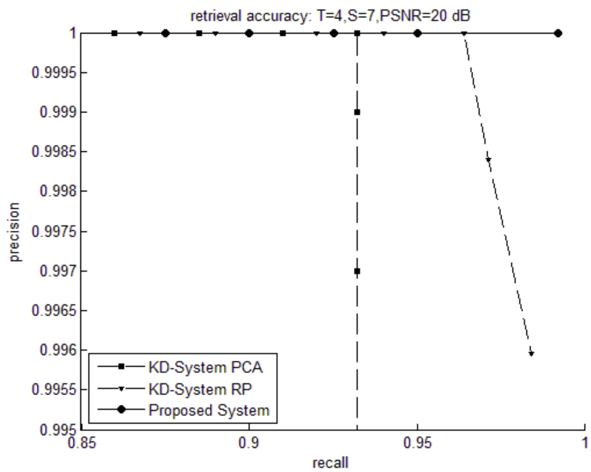


Fig. 4. Precission-Recall curves for three video-retrieval systems: KD-System PCA [12], KD-System RP [13], and Proposed Method.

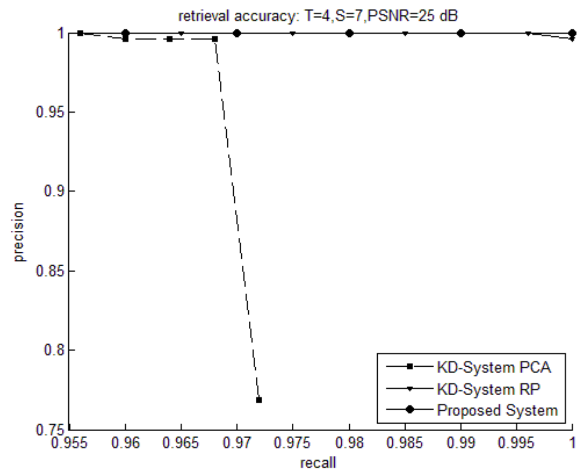


Fig. 5. Precission-Recall curves for three video-retrieval systems: KD-System PCA [12], KD-System RP [13], and Proposed Method.

$S = 7$, and $PSNR = 20\ dB$. The method proposed in [12] performs worse than the one in [13]. Furthermore, the proposed sparse Bayesian retrieval method performs better than the method in [13]. As shown in Fig. 4 its precision-recall curve is $precision = 1$ for $recall \leq 0.98$

Finally in Fig. 5 the three systems are compared for the case $T = 4$, $S = 7$ and $PSNR = 25\ dB$. Again the method proposed in [12] performs worse than the one in [13]. Furthermore, the proposed sparse Bayesian retrieval method performs better than the method in [13]. As shown in Fig. 5 its precision-recall curve is $precision = 1$ since the threshold values for all relevant clips are smaller than the corresponding to nonrelevant clip. The same behavior is observed for higher PSNR levels. When more frames are included in the query, that is when $S = 15$, the precision-recall curve for the proposed method is $precision = 1$.

6. CONCLUSION

In this paper we have developed a robust and efficient system for video retrieval, based on the use of sparse representation, compressive sensing and Bayesian modeling of the video retrieval problem. Experimental results demostrate that the proposed method performs better than existing state-of-art systems and also its robustness against noise. Work to tackle the problems of occlusions and missing frames is already in progress.

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