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Optical Implementation of Iterative Algorithms for Image Restoration.

An Electrical Engineering
Master's Project Report

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1 Introduction

Iterative algorithms have been used to achieve image restoration [1],[2]. Such algorithms range from the simple Jacobi algorithm to more refined ones. Their implementation by purely numerical methods has proved to be time consuming due to the large amount of computations necessary between successive iterations.

In view of the above, TV-optical implementations using incoherent optical feedback systems and analog addition and substraction of video signals have been suggested [3]. However, only limited practical implementations have been demonstrated [4].

A modification to the above system has also been demonstrated were an optical-digital approach is used [5]. The strategy in this case is to continue to perform the 2-D convolution optically but then convert the analog TV signal to digital format and perform the necessary additions and substractions in a minicomputer.

As it will explain, the vast majority of the difficulties experienced with the two implementations mentioned above, arise from the fact that they are hybrid systems using TV-optical and optical-digital techniques.

In view of these facts, we propose in this report a new totally optical approach to image restoration using iterative algorithms.

2 Iterative Algorithms for Image restoration

Several different iterative algorithms based on the method of successive approximations have been applied to the image restoration problem, the main difference among them being the speed of convergence.

Since by using optical processing techniques a high speed of computation is achieved, the convergence rate becomes less important and the simplicity of implementation becomes the prime consideration.

In view of the above, we will only concern ourselves here with the two most basic iterative algorithms of this type and will only mention their characteristics for completeness. A more detailed treatment of these and other algorithms can be found in references [1] and [2].

2.1 The Jacobi Method

The Jacobi method, also known as the simultaneous displacements method, is the simplest of this class of iterative algorithms used for image restoration.

Here the original picture represented by a vector F has been downgraded by a linear defect represented by the matrix D resulting in a blurred image represented by the vector G, according to

$$G = D \cdot F \tag{1}$$

The Jacobi method is then to find F by using the iterative equation:

$$F^{(k+1)} = F^{(k)} + \beta(G - D \cdot F^{(k)}) \tag{2}$$

with the initial solution usually chosen to be equal to

$$F^{(0)} = G \tag{3}$$

where (k) denotes the k-th iteration step and β the relaxation parameter that controls the convergence and the rate of convergence of the iteration.

For a space invariant distortion equation (2) can also be written in the spatial frequency domain as:

$$F^{(k+1)}(f_x,f_y) = \beta G(f_x,f_y) + F^{(k)}(f_x,f_y)\,(1 - \beta D(f_x,f_y)) \tag{4}$$

where f_x, f_y denote spatial frequencies.

As it can be easily shown, the sufficient condition for convergence for iteration (4) is given by

$$| 1 - \beta D(f_x, f_y) | < 1 \tag{5}$$

A clear deficiency of this algorithm is that the defect transfer function $D(f_x, f_y)$ cannot be negative, since condition (5) cannot be satisfied in this case. If, however, $D(f_x, f_y)$ which is normalized, satisfies the condition

$$0 \le D(f_x, f_y) \le 1 \tag{6}$$

condition (5) results in

$$0 \le \beta \le 2 \tag{7}$$

2.2 The Jacobi Method with Reblurring

To overcome the restriction requiring the defect transfer function to be positive, a reblurring procedure can be performed. In other words, the already blured image vector G and the distortion matrix D are intentionally degraded by another distortion represented by the matrix D^T.

Under these circumstances the algorithm now becomes:

$$F^{(k+1)} = F^{(k)} + \beta D^T (G - D \cdot F^{(k)}) \tag{8}$$

with the initial guess solution chosen to be

$$F^{(0)} = D^T \cdot G \tag{9}$$

Writing equation (8) in the spatial frequency domain we obtain

$$F^{(k+1)}(f_x, f_y) = \beta D^*(f_x, f_y) G(f_x, f_y) + F^{(k)}(f_x, f_y)(1 - \beta | D(f_x, f_y) |^2) \tag{10}$$

The sufficient condition for convergence now becomes

$$| 1 - \beta | D(f_x, f_y) |^2 | < 1 \tag{11}$$

which is satisfied for

$$0 \le \beta \le 2 \tag{12}$$

since it is always true that

$$0 \leq | D(f_x, f_y) |^2 \leq 1 \tag{13}$$

$D(f_x, f_y)$ is clearly not restricted in sign in this case.

3 Hybrid Optical Implementations

As mentioned in our introduction several attempts have been made to implement the algorithms described in the previous section by means of hybrid optical systems. In this section we will examine these systems and point out the difficulties encountered in their implementation.

3.1 The TV-Optical Approach

A TV-optical approach for image restoration using iterative algorithms has been proposed by Maitre [3]. Several algorithms were considered taking into consideration the ease of implementation by TV-optical means as well as the speed of convergence.

Among the algorithms considered are the Jacobi method, the Gauss-Seidel method (also known as the successive displacement method), the steepest descent method, and others.

From the different methods considered, the Jacobi method was selected because of its simplicity despite it's slowness of convergence as this is compensated by the relative high speed of the optical processor when compared to the pure numeric one.

The resulting TV-optical system is depicted in figure 1.

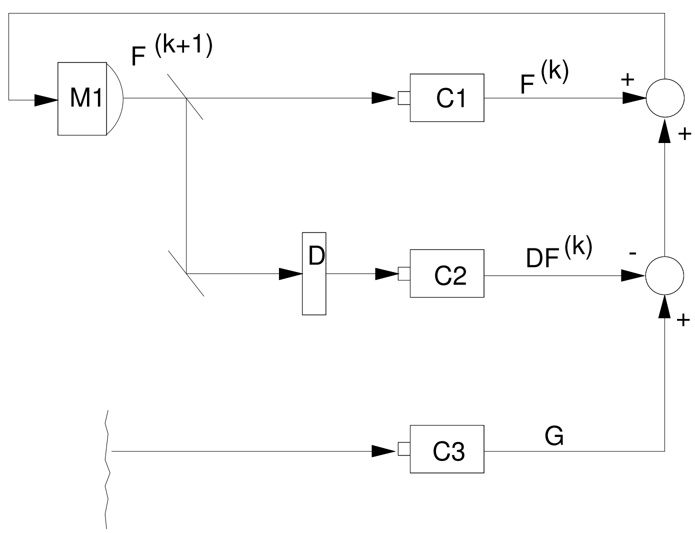

Fig. 1: Hybrid TV-Optical implementation of the Jacobi Algorithm.

In this figure M1 denotes a monitor and C1, C2 and C3 denote cameras 1 through 3.

Experiments were performed by Ferrano and Maitre [4]. The defect D was electronically simulated thereby not requiring the use of the camera C2. The defect used corresponds to a low pass filter applied to the video signal and therefore is unidimensional.

In order to insure that the convergence conditions (i.e. positivity and saturation constraints) were met, the video signal was subject to hard-clipping before displaying it on the CRT. Although not mentioned in [4], in order to be able to perform the required additions all cameras and the monitor must have been synchronized.

The implementation of a 2-D distortion would require the use of the camera C2. A major difficulty in this case is the requirement that both cameras (C1 and C2) be perfectly aligned, so that perfect registration of the two video signals occurs. Furthermore, C1 and C2 must have identical transfer function in order to be able to achieve a true substraction of $F^{(k)}$ and $DF^{(k)}$. In other words, C1 and C2 must be identically matched cameras.

The experimental results shown in [4] indicate a noticeable increase in image quality due to the restoration and a speed of convergence of less than one second.

3.2 The Optical-Digital Approach

A modification to the TV-optical approach has been demonstrated by Matsuoka et. al. [5]. The system is based on the same iterative algorithm but the implementation differs in that the video signals are converted to digital format for processing. A schematic of such a system is presented in figure 2.

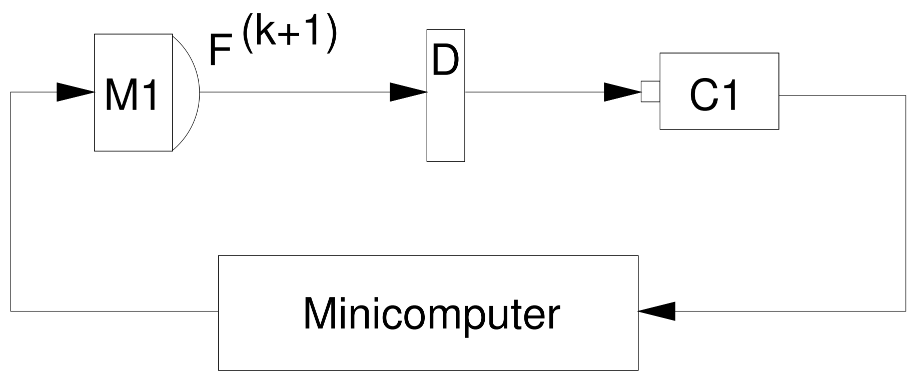

Fig. 2: Optical digital setup

Here a minicomputer is used to process the information received from the camera. The processed output is applied to the monitor, and is subsequently passed through the defect D before reaching the camera as $DF^{(k)}$.

In the minicomputer a copy of the starting blurred image G is kept in memory along with the latest processed image $F^{(k)}$. With this information the new processed image $F^{(k+1)}$ is calculated and the same procedure repeated until a satisfactory output is achieved. A similar procedure can also be used to implement the Jacobi method with reblurring using the same physical setup system.

Experiments were demonstrated with this type of system. To produce a blurred image, the optical system is presented with the original image and the output of the camera (the original blurred by defect D) is then taken as the blurred input image. In order to avoid the need for synchronization between camera and monitor, the images acquired by the camera were averaged digitally. This process takes 75 seconds approximately per 144X144 pixel image. Also, non-linearities introduced by the camera and monitor were compensated digitally by the minicomputer by applying correction tables stored in memory. It takes 230 msec approximately to perform this compensation per 144X144 pixel image. The total time required per iteration is between 70 to 150 seconds depending on the algorithm used and the number of pixels in the image to be processed.

An advantage of this system over the TV-optical system is that it can be used to implement both the Jacobi method (Eq. (2)), and the Jacobi method with reblurring (Eq. (8)), thus allowing it to overcome the positivity constraint imposed on $D(f_x, f_y)$ in Eq. (6). A disadvantage of this system is of course the much larger convergence time as compared to the TV-optical system. It is however, claimed to be still much faster than a purely numerical approach.

Since the processing time is determined largely by the image acquisition time, it is suggested in reference [5] that a hardware modification to the setup used in the experiments, employing a TV rate A/D converter, could allow the acquisition of a 512X512 pixel image within 1 second by averaging 32 frames. If such system is developed the processing time would of course be dramatically improved.

4 A Completely Optical Approach

In recent years there has been research in the field of coherent optical image amplifiers [6] and coherent optical feedback systems [7]. One of such works has demonstrated an active coherent optical feedback system suitable for image transmission [7]. By using a phase-conjugating mirror (PCM) as an amplifying element, experiments where performed showing the feasibility of using this system for image transmission.

In these experiments, the input beam is bidimensionally amplitude modulated by a transparency. The image containing beam is then fed to the system that includes a PCM, a beam splitter and several mirrors to achieve an amplifier with negative feedback configuration. Registration of the beams is achieved by controlling the optical path lengths.

Several setups were tried. One of these setups uses polarized beams to obtain a unidirectional ring with coherent light circulating in only one direction. This system is depicted in figure 3.

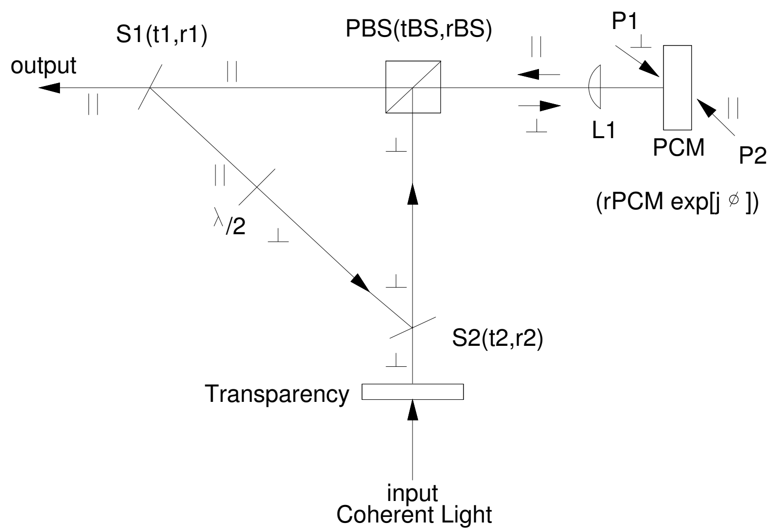

Fig. 3: Coherent optical feedback system

In this figure PBS denotes a polarizing beam splitter, S denotes a mirror and t and r respectively the transmission and reflection coefficients. Furthermore, the parallel and perpendicular symbols denote polarization respectively, to the plane of the paper.

We modify this system to perform image restoration by means of the Jacobi algorithm with and without reblurring. To acquire a transparency of the distorted image so that it can be illuminated by coherent light, several methods can be used. On of such methods is the employment of liquid crystal display (LCD) as a spatial light modulator. An analysis of this type of modulator can be found in reference [8].

In order to describe the proposed system for iterative image restoration we need to introduce the concept of phase conjugate optics and establish notation. This is the objective of the following section 4.1.

4.1 Phase Conjugate Optics

In this section we will digress a little to briefly describe the concept of phase conjugate optics. Here we will only mention the basic subjects required to analyze our optical system for image restoration. A more detailed treatment of the topic can be found for example in reference [9].

4.1.1 Phase Conjugate systems

Phase conjugate optics refers to systems where for a given monochromatic optical field input such as

$$E_1(x,y,z,t) = Re[\psi(x,y)e^{j(\omega t - kz)}] \tag{14a}$$

produce an output of the form

$$E_2(x,y,z,t) = Re[\psi^*(x,y)e^{j(\omega t + kz)}] \tag{14b}$$

with

$$k = \frac{2\pi}{\lambda}$$

where x,y,z,t are the space and time coordinates, ω is the angular frequency, $\psi(\cdot)$ is a complex function of space and λ the wavelength.

From the above expressions it is clear that E_2 can be obtained from E_1 by replacing the spatial part (i.e. the complex amplitude) by its complex conjugate but leaving the factor exp[jωt] intact. Under these circumstances, E_2 is called the phase conjugate replica of E_1.

This relationship between E_1 and E_2 implies that their wavefronts coincide everywhere, the only difference being the fact that they propagate in opposite directions.

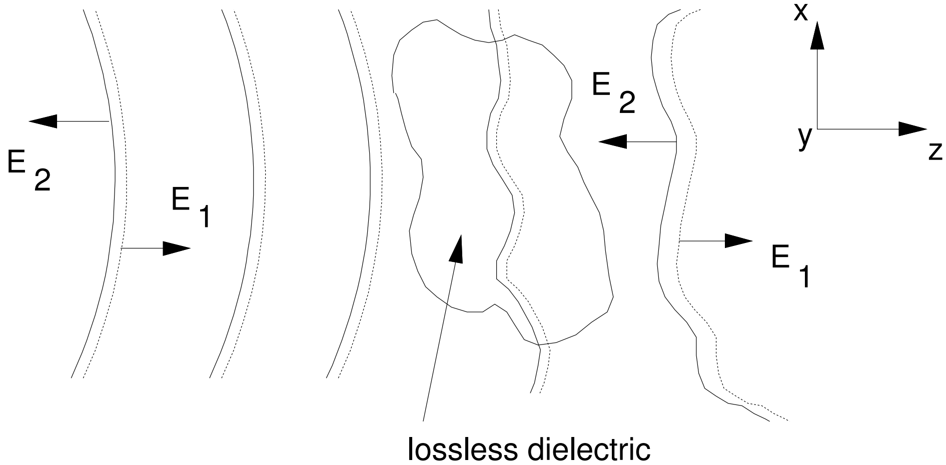

Fig. 4: Distortion correction theorem.

An important consequence of this, is the *distortion correction theorem* (depicted in figure 4). It basically states that if E_1 propagates through a lossless arbitrary dielectric medium, then, if we generate its phase conjugate replica E_2 at some point in space, E_2 will propagate backwards through the lossless dielectric medium remaining a phase conjugate of E_1 everywhere. In other words, any phase distortion introduced by the dielectric will be undone when E_2 propagates backwards through it.

As an example, suppose E_1 goes through a lens and then E_2 is generated. Then, the effect of the lens on E_1 will be reversed on E_2 after it passes through it.

4.1.2 Phase Conjugation in the Spatial Frequency Domain

We will now analyze the effects of phase conjugation on the spatial frequency domain. The reason for this will become apparent when we carry out the optical analysis of our image restoration systems.

From now on, we will use the complex notation to refer to scalar light waves and will only concern ourselves with their complex amplitude. This is done for dealing with simpler expressions.

Suppose the input wave to a phase conjugate system has complex amplitude given by $U_{in}(x,y)$ at the z=0 plane, with 2-D Fourier transform $U_{in}(f_x,f_y)$. Then its phase conjugate replica and corresponding 2-D Fourier transform will be given by the expressions

$$U_{\text{out}}(x,y) = U^*_{\text{in}}(x,y) \tag{15}$$

$$U_{\text{out}}(f_x,f_y) = U^*_{\text{in}}(-f_x,-f_y) \tag{16}$$

An important case arises when $U_{in}(x,y)$ is real. Under these circumstances, it's Fourier transform is said to be *Hermitian* and therefore satisfies

$$U_{\text{in}}(f_x,f_y) = U^*_{\text{in}}(-f_x,-f_y) \tag{17}$$

Another important case for us, is when $U_{in}(x,y)$ is an even function. Under these circumstances its Fourier transform is also even and will comply with

$$U^*_{\text{in}}(f_x,f_y) = U^*_{\text{in}}(-f_x,-f_y) \tag{18}$$

These expressions will prove useful in the following sections.

4.1.3 Phase Conjugating Mirror (PCM)

We will now briefly describe the operation of the PCM. Again, we will not give here a detailed treatment of the topic, but only the basic concept. As before a more detailed description can be found for example in reference [9].

A PCM is an optical system that employs a nonlinear medium (such as a crystal) to generate the phase conjugate replica of an input light wave using a four wave mixing configuration. Figure 5 shows the basic setup.

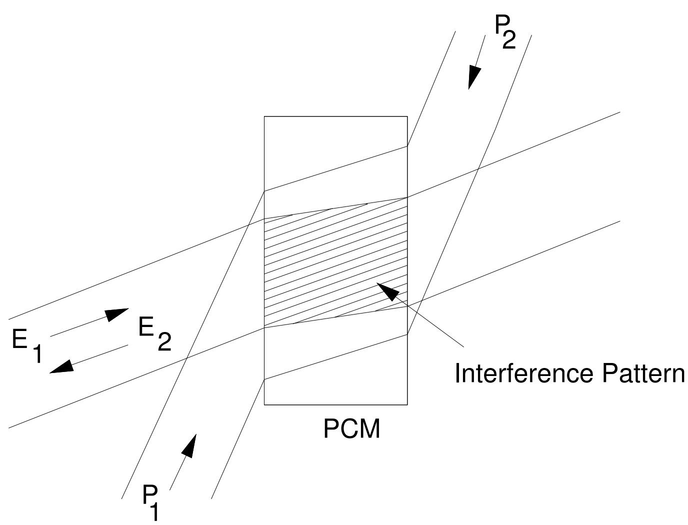

Fig. 5: Four wave mixing configuration

Here the input beam E_1 and the writing pump beam P_1, form an interference pattern that in turn creates a refractive index grating due to the photorefractive effect. The reading pump beam P_2, then scatters from the refractive index grating to form the phase conjugate replica of E_1 multiplied by a complex constant factor and labeled here as E_2.

If we define $U_1(x,y,z)$ and $U_2(x,y,z)$ as the complex amplitudes of E_1 and E_2 respectively, we can write:

$$U_2(x,y,z) = r_{\mathrm{PCM}} e^{j\phi} U_1^*(x,y,z) \tag{19}$$

Depending on the material of the crystal, its physical dimensions and the polarization of the beams, the factor r_{PCM} can be made to be more than unity, thus creating an amplified phase conjugate version of the input wave. An amplification factor of up to 4000 has been obtained in reference [10] by using a barium titanate ($BaTiO_3$) crystal.

4.2 Image Restoration system 1

We are finally in a position to formulate an optical system that will perform image restoration by applying the algorithms described in section 2. We first propose a system which implements the Jacobi algorithm for a distortion with a real spatial impulse response D(x,y).

4.2.1 Optical Analysis

Lets refer ourselves to the system depicted in figure 6.

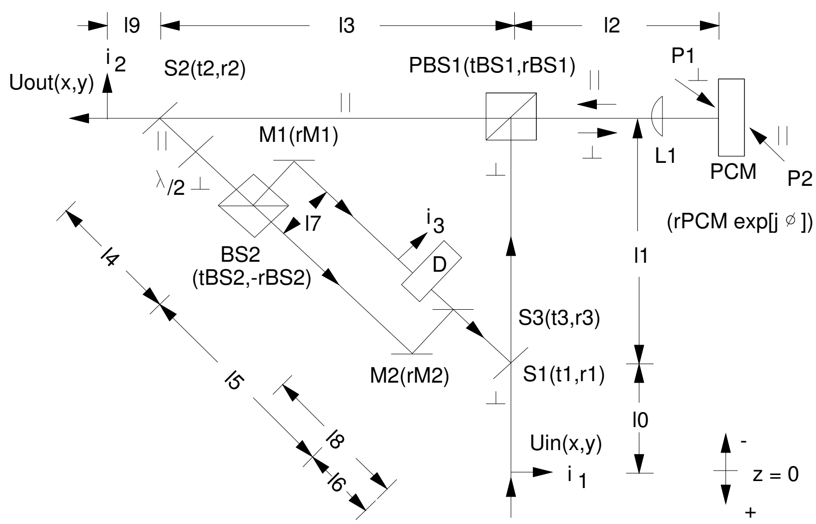

Fig. 6: Optical Image restoration system 1

We define by $U_{in}(x,y)$ the complex amplitude of a plane wave modulated by the 2-D amplitude distribution (transparency) at the input plane i_1.

$$U_{\mathrm{in}}(x,y) = U_o(x,y) e^{j\phi_o} \tag{20}$$

In other words, $U_{in}(x,y)$ is composed by a real function $U_o(x,y)$ multiplied by a constant phase factor.

In the spatial frequency domain, this expression becomes

$$U_{\text{in}}(f_x,f_y) = U_o(f_x,f_y)e^{j\phi_o} \tag{21}$$

We also define D as a distortion with real spatial impulse response given by D(x,y) with 2-D Fourier transform $D(f_x,f_y)$. Since both D(x,y) and $U_o(x,y)$ are real, their spatial frequency counterparts are *Hermitian* and behave according to (17).

Finally, we define $U_{out}(x,y)$ as the complex amplitude of the output wave at the output plane i_2, with 2-D Fourier transform $U_{out}(f_x,f_y)$. All other components have transmission and/or reflection coefficients as shown.

A thin lossless lens, denoted by L1 in figure 6, is used on the PCM's bidirectional arm to focus the beam unto the crystal. This lens is ignored on the following analysis because it has no effect on the output because of the *distortion correction theorem*.

In the following analysis, we find a transfer function for the system of figure 6 by adding the output components produced by infinite passes of the light through the system. In doing so, we also use the Fresnel approximation for the free space transfer function given by

$$H(f_x,f_y) = e^{jkz}e^{-j\pi\lambda z|f|^2}$$

where

$$|f|^2 = f_x^2 + f_y^2$$

First Pass:

$$U_{\text{out}}(f_x,f_y) = t_1 t_2 r_{\text{BS1}} t_{\text{BS1}} r_{\text{PCM}} e^{j\left\{\phi + k\left(l_o + l_1 - l_3 - l_9\right)\right\}} e^{-j\pi\lambda\left(l_o + l_1 - l_3 - l_9\right)|f|^2} U^*_{\text{in}}(f_x,f_y)$$
$$= U_1(f_x,f_y)$$

Second Pass:

$$U_{\text{out}}(f_x,f_y) = U_1(f_x,f_y) + t_1 t_2 r^2_{\text{BS1}} t^2_{\text{BS1}} r^2_{\text{PCM}} r_1 r_2 \{t_{\text{BS2}} r_{\text{M2}} r_3 - r_{\text{BS2}} r_{\text{M1}} t_3 D(f_x,f_y)\}$$
$$e^{jk\left(l_4 + l_5 + l_6 + l_7 - l_o - l_9\right)} e^{-j\pi\lambda\left(l_4 + l_5 + l_6 + l_7 - l_o - l_9\right)|f|^2} U_{\text{in}}(f_x,f_y)$$
$$= U_1(f_x,f_y) + U_2(f_x,f_y)$$

Third Pass:

$$U_{\text{out}}(f_x,f_y) = U_1(f_x,f_y) + U_2(f_x,f_y) + t_1 t_2 r_{\text{BS1}}^3 t_{\text{BS1}}^3 r_{\text{PCM}}^3 r_1^2 r_2^2$$

$$[t_{\text{BS2}} r_{\text{M2}} r_3 - r_{\text{BS2}} r_{\text{M1}} t_3 D(f_x,f_y)]^2$$

$$e^{j\left\{\phi + k\left(l_o + l_1 - l_3 - l_9\right)\right\}} e^{-j\pi\lambda\left(l_o + l_1 - l_3 - l_9\right)|f|^2} U_{\text{in}}^*(f_x,f_y)$$

For correct imaging we need the system's geometry to satisfy:

$$l_o + l_1 = l_3 + l_9 = l_3 + l_4 + l_5 + l_6 + l_7 - l_8 \tag{22}$$

and

$$l_8 = l_o \tag{23}$$

In this case, after the Nth Pass we obtain

$$\begin{aligned} U_{\text{out}}(f_x,f_y) = t_1 t_2 r_{\text{BS1}} t_{\text{BS1}} r_{\text{PCM}} \{ & e^{j\phi} U_{\text{in}}^*(f_x,f_y) \\ & + r_{\text{BS1}} t_{\text{BS1}} r_1 r_2 r_{\text{PCM}} [t_{\text{BS2}} r_{\text{M2}} r_3 - r_{\text{BS2}} r_{\text{M1}} t_3 D(f_x,f_y)] U_{\text{in}}(f_x,f_y) \\ & + r_{\text{BS1}}^2 t_{\text{BS1}}^2 r_1^2 r_2^2 r_{\text{PCM}}^2 [t_{\text{BS2}} r_{\text{M2}} r_3 - r_{\text{BS2}} r_{\text{M1}} t_3 D(f_x,f_y)]^2 e^{j\phi} U_{\text{in}}^*(f_x,f_y) \\ & + \ldots \} \end{aligned}$$

By setting

$$\beta = \frac{r_{\text{BS2}} r_{\text{M1}} t_3}{t_{\text{BS2}} r_{\text{M2}} r_3}$$

$$r_{\text{S1}} = t_{\text{BS1}} r_{\text{BS1}} r_1 r_2 t_{\text{BS2}} r_{\text{M2}} r_3$$

we obtain

$$U_{\text{out}}(f_x,f_y) = t_1 t_2 r_{\text{BS1}} t_{\text{BS1}} r_{\text{PCM}} \{ e^{j\phi} U_{\text{in}}^*(f_x,f_y)$$
$$+ r_{\text{S1}} r_{\text{PCM}} [1 - \beta D(f_x,f_y)] U_{\text{in}}(f_x,f_y)$$
$$+ r_{\text{S1}}^2 r_{\text{PCM}}^2 [1 - \beta D(f_x,f_y)]^2 e^{j\phi} U_{\text{in}}^*(f_x,f_y)$$
$$+ \ldots \}$$

$$\frac{U_{\text{out}}(f_x,f_y)}{U_{\text{o}}(f_x,f_y)} = t_1 t_2 r_{\text{BS1}} t_{\text{BS1}} r_{\text{PCM}} \{ (1 + r_{\text{S1}}^2 r_{\text{PCM}}^2 [1 - \beta D(f_x,f_y)]^2 + \ldots) e^{j(\phi - \phi_o)}$$
$$+ r_{\text{S1}} r_{\text{PCM}} [1 - \beta D(f_x,f_y)] (1 + r_{\text{S1}}^2 r_{\text{PCM}}^2 [1 - \beta D(f_x,f_y)]^2 + \ldots) e^{j\phi_o} \}$$

if the condition

$$r_{\text{S1}}^2 r_{\text{PCM}}^2 [1 - \beta D(f_x,f_y)]^2 < 1 \tag{24}$$

is satisfied, we obtain

$$\frac{U_{\text{out}}(f_x,f_y)}{U_{\text{o}}(f_x,f_y)} = \frac{t_1 t_2 r_{\text{BS1}} t_{\text{BS1}} r_{\text{PCM}}}{1 - r_{\text{S1}}^2 r_{\text{PCM}}^2 [1 - \beta D(f_x,f_y)]^2} \left\{ e^{j(\phi - \phi_o)} + r_{\text{S1}} r_{\text{PCM}} [1 - \beta D(f_x,f_y)] e^{j\phi_o} \right\}$$

Therefore the intensity transfer function is equal to

$$T(f_x,f_y) = \left| \frac{U_{\text{out}}(f_x,f_y)}{U_{\text{o}}(f_x,f_y)} \right|^2$$

or

$$T(f_x,f_y) = \left(\frac{t_1 t_2 r_{\text{BS1}} t_{\text{BS1}} r_{\text{PCM}}}{1 - r_{\text{S1}}^2 r_{\text{PCM}}^2 [1 - \beta D(f_x,f_y)]^2} \right)^2 \{ 1 + r_{\text{S1}}^2 r_{\text{PCM}}^2 [1 - \beta D(f_x,f_y)]^2$$
$$+ 2 r_{\text{S1}} r_{\text{PCM}} [1 - \beta D(f_x,f_y)] \cos(\phi - 2\phi_o) \}$$

Clearly from the above expression in order to get the desired result, it is required that

$$r_{\text{S1}} r_{\text{PCM}} = 1 \text{ and } \quad \phi = 2\phi_o \tag{25}$$

Then $T(f_x,f_y)$ becomes equal to

$$T(f_x,f_y) = \left(\frac{t_1 t_2 r_{BS1} t_{BS1} r_{PCM}}{\beta D(f_x,f_y)} \right)^2 = \left(\frac{t_1 t_2}{r_1 r_2 r_{BS2} r_{M1} t_3 D(f_x,f_y)} \right)^2 \quad (26)$$

Therefore, as can be seen from (26), provided that the conditions of (22), (23) and (25) are met and the convergence condition expressed by (24) is satisfied, this system has a transfer function which is the inverse of $D(f_x,f_y)$ times a constant gain factor. This means that the output image will be a restored version of the input image with increased or decreased overall intensity, depending on the value of the gain factor. Notice from (26) that if $t_1 t_2 r_{BS1} t_{BS1} r_{PCM} = \beta$, then the system performs the Jacobi algorithm exactly as it is expressed in (4), that is, the resulting gain factor is equal to unity.

4.2.2 Geometry Analysis

We have seen that in order to obtain the desired results with the system of figure 6 it is a requirement that the system's geometry be consistent with equations (22) and (23). Therefore we need to analyze the system's geometry to insure that this is the case.

In order to perform this analysis we shall require to find the mathematical relationship that exists between the sides of the system. Let us then refer to figure 7, in which the system's geometry is depicted.

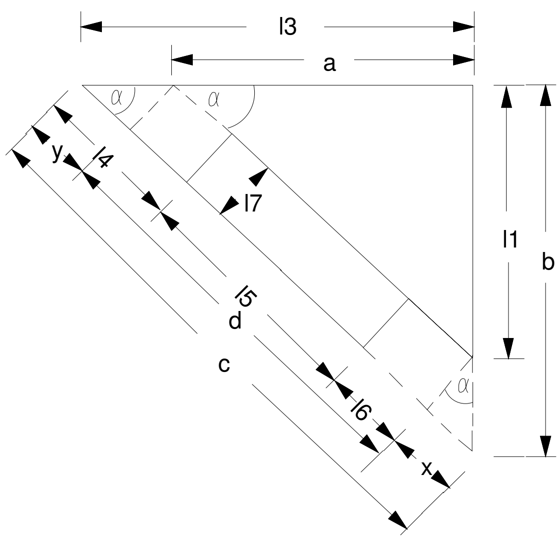

Fig. 7: System 1 geometry

By inspecting figure 7 we can write:

$$c^2 = l_3^2 + b^2 \ ; \quad b = l_3 \tan\alpha$$

$$d^2 = l_1^2 + a^2 \ ; \quad a = l_1 \cot\alpha$$

so that,

$$c^2 = l_3^2(1+\tan^2\alpha) = l_3^2\sec^2\alpha \tag{27}$$

$$d^2 = l_1^2(1+\cot^2\alpha) = l_1^2\operatorname{cosec}^2\alpha \tag{28}$$

From figure 7 we can also write:

$$c = L + x = L + l_7\tan\alpha \Rightarrow c^2 = L^2 + 2Ll_7\tan\alpha + l_7^2\tan^2\alpha \tag{29}$$

$$d = L - y = L - l_7\cot\alpha \Rightarrow d^2 = L^2 - 2Ll_7\cot\alpha + l_7^2\cot^2\alpha \tag{30}$$

where $L = l_4 + l_5 + l_6$

Substituting (27) in (29) and (28) in (30) we get:

$$L^2 + 2Ll_7\tan\alpha + l_7^2\tan^2\alpha = l_3^2\sec^2\alpha$$

$$L^2 - 2Ll_7\cot\alpha + l_7^2\cot^2\alpha = l_1^2\operatorname{cosec}^2\alpha$$

which can also be written as

$$L^2\cos^2\alpha + 2Ll_7\sin\alpha\cos\alpha + l_7^2\cos^2\alpha = l_3^2 \tag{31}$$

$$L^2\sin^2\alpha - 2Ll_7\sin\alpha\cos\alpha + l_7^2\sin^2\alpha = l_1^2 \tag{32}$$

Finally by adding (31) and (32) we obtain:

$$L^2 + l_7^2 = l_3^2 + l_1^2 \tag{33}$$

which is the relationship between the sides of the system's geometry. With this result, we can now see if the geometry requirements, namely equations (22) and (23) can be achieved.

To do this, let us derive the expressions for l_o and l_9. From (22) and (23) we find that

$$l_o = \frac{L - l_1 + l_3 + l_7}{2} \tag{34}$$

$$l_9 = \frac{L + l_1 - l_3 + l_7}{2} \quad (35)$$

Since both l_o and l_9 must be positive numbers, then it is easy to see from equations (34) and (35) that the following inequalities must be satisfied

$$l_1 - l_3 < L + l_7$$

and

$$l_3 - l_1 < L + l_7$$

This reduces to

$$| l_1 - l_3 | < L + l_7 \quad (36)$$

Knowing that L and l_7 are both positive quantities, Eq. (36) can be rewritten as

$$(l_1 - l_3)^2 < (L + l_7)^2$$

or

$$l_1^2 + l_3^2 - 2l_1l_2 < L^2 + l_7^2 + 2Ll_7 \quad (37)$$

Applying (33) to (37) we find that

$$-l_1l_2 < Ll_7 \quad (38)$$

Since equation (38) is always satisfied, it follows that the geometry requirements (22) and (23) can be realized with the proposed configuration.

4.3 Image Restoration system 2

We have seen that the system described in the preceding sections, performs image restoration by applying the Jacobi method. However, the type of distortion it can undo is restricted to be a real function of space.

In the following sections, we describe a modification to system 1 that will allow the restoration of images degraded by distortions which are complex functions of space. To achieve this goal, a system that employs two PCMs is required. Such a system is depicted in figure 8.

4.3.1 Optical Analysis

In this section we calculate the transfer function of the system of figure 8 by the same method employed before with system 1. All variables are defined the same way as before, except that D(x,y) is now a complex function.

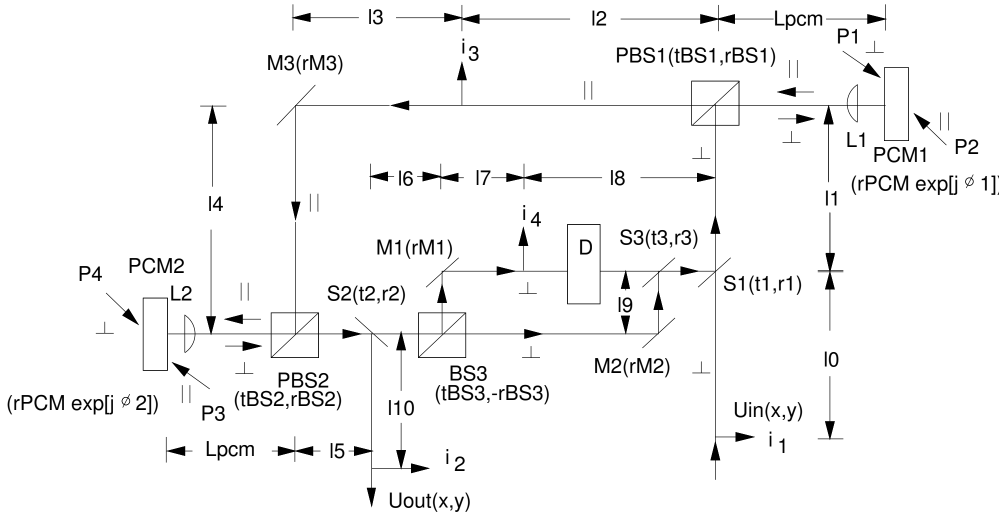

Fig. 8: Optical image restoration system 2

To simplify matters, we will assume that the imaging conditions are satisfied. That is

$$l_o + l_1 = l_2 \tag{39}$$

$$l_3 + l_4 = l_5 + l_6 + l_7 + l_9 \tag{40}$$

$$l_3 + l_4 = l_5 + l_{10} \tag{41}$$

and

$$l_8 = l_o \tag{42}$$

Under these circumstances we repeat the same procedure that was used with system 1. More specifically, the output function is given by the following expressions

First Pass:

$$U_{\text{out}}(f_x,f_y) = t_1 r_2 r_{\text{BS1}} t_{\text{BS1}} r_{\text{BS2}} t_{\text{BS2}} r_{\text{M3}} r_{\text{PCM1}} r_{\text{PCM2}} e^{j(\phi_2 - \phi_1)} U_{\text{in}}(f_x,f_y)$$

$$= U_1(f_x,f_y)$$

Second Pass:

$$U_{\text{out}}(f_x,f_y) = U_1(f_x,f_y) + t_1 r_2 r_{\text{BS1}}^2 t_{\text{BS1}}^2 r_{\text{BS2}}^2 t_{\text{BS2}}^2 r_{\text{M3}}^2 r_{\text{PCM1}}^2 r_{\text{PCM2}}^2 r_1 t_2$$

$$\{t_{\text{BS3}} r_{\text{M2}} r_3 - r_{\text{BS3}} r_{\text{M1}} t_3 D(f_x,f_y)\} e^{j2(\phi_2 - \phi_1)} U_{\text{in}}(f_x,f_y)$$

$$= U_1(f_x,f_y) + U_2(f_x,f_y)$$

By setting

$$\beta = \frac{r_{\text{BS3}} r_{\text{M1}} t_3}{t_{\text{BS3}} r_{\text{M2}} r_3}$$

$$r_{\text{S1}} = t_{\text{BS1}} r_{\text{BS1}} t_{\text{BS2}} r_{\text{BS2}} r_{\text{M3}} t_{\text{BS3}} r_{\text{M2}} r_1 r_3 t_2$$

$$r_{\text{PCM}} = r_{\text{PCM1}} r_{\text{PCM2}}$$

we obtain at the Nth Pass:

$$U_{\text{out}}(f_x,f_y) = t_1 r_2 r_{\text{BS1}} t_{\text{BS1}} r_{\text{BS2}} t_{\text{BS2}} r_{\text{M3}} r_{\text{PCM}} e^{j(\phi_2 - \phi_1)} U_{\text{in}}(f_x,f_y)$$

$$\{1 + r_{\text{S1}} r_{\text{PCM}} [1 - \beta D(f_x,f_y)] e^{j(\phi_2 - \phi_1)}$$

$$+ r_{\text{S1}}^2 r_{\text{PCM}}^2 [1 - \beta D(f_x,f_y)]^2 e^{j2(\phi_2 - \phi_1)} + \ldots\}$$

If the condition

$$\left| r_{\text{S1}} r_{\text{PCM}} [1 - \beta D(f_x,f_y)] e^{j(\phi_2 - \phi_1)} \right| < 1 \qquad (43)$$

is satisfied, we obtain

$$\frac{U_{\text{out}}(f_x,f_y)}{U_{\text{o}}(f_x,f_y)} = \frac{t_1 r_2 r_{\text{BS1}} t_{\text{BS1}} r_{\text{BS2}} t_{\text{BS2}} r_{\text{M3}} r_{\text{PCM}} e^{j\left(\phi_2 - \phi_1 - \phi_o\right)}}{1 - r_{\text{S1}} r_{\text{PCM}} [1 - \beta D(f_x,f_y)] e^{j(\phi_2 - \phi_1)}}$$

To get the desired result it is required that

$$r_{\text{S1}} r_{\text{PCM}} = 1 \ \text{ and } \quad \phi_2 = \phi_1 \tag{44}$$

Finally,

$$T(f_x,f_y) = \left| \frac{U_{\text{out}}(f_x,f_y)}{U_{\text{o}}(f_x,f_y)} \right|^2 = \left| \frac{t_1 r_2 r_{\text{BS1}} t_{\text{BS1}} r_{\text{BS2}} t_{\text{BS2}} r_{\text{M3}} r_{\text{PCM}} e^{j\phi_o}}{\beta D(f_x,f_y)} \right|^2$$

or

$$T(f_x,f_y) = \left(\frac{t_1 r_2}{r_1 t_2 r_{\text{BS3}} r_{\text{M1}} t_3 \left| D(f_x,f_y) \right|} \right)^2 \tag{45}$$

So, once again, provided that conditions (39), (40), (41) (42) and (44) are met and that the convergence condition expressed by (43) is satisfied, this system has a transfer function which is the inverse of $D(f_x,f_y)$, multiplied by a constant gain factor.

4.3.2 Geometry Analysis

Once more we analyze the system geometry to ensure that it meets the geometry requirements which are given by (39), (40), (41) and (42). With this geometry, the relationship between the sides of the system, is easily seen from figure 9.

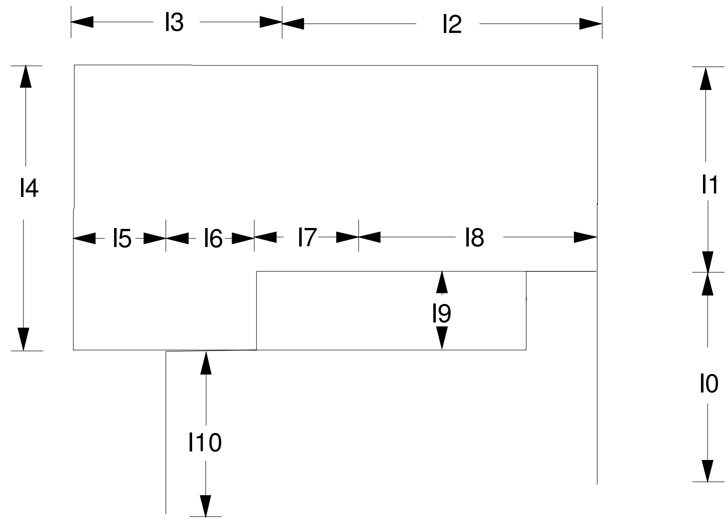

Fig. 9: System 2 geometry

We can write:

$$l_9 + l_1 = l_4 \tag{46}$$

$$l_3 + l_2 = l_5 + l_6 + l_7 + l_8 \tag{47}$$

By combining (40) with (47) we get

$$l_2 - l_4 = l_8 - l_9 \tag{48}$$

Applying (39) and (46) to (48) we obtain

$$l_o + l_1 - l_9 - l_1 = l_8 - l_9$$

which reduces to equation (42). It follows that the system's geometry is consistent with the required imaging conditions. To find the correct distances to the input and output planes, l_o and l_{10} given respectively by equations (39) and (41), can be used.

4.4 Image Restoration system 3

So far we have discussed two optical systems that implement the Jacobi method to achieve image restoration. However, as it was discussed in section 2, this method does not allow $D(f_x,f_y)$ to have negative values. To circumvent this deficiency the Jacobi method with reblurring was introduced.

In this section propose a third system that implements the Jacobi method with reblurring by employing three PCMs. One of the PCMs is used to obtain the required $|D(f_x,f_y)|^2$ term by placing the distortion in its bidirectional arm. To get the desired result it is required that D(x,y) be an even function, so that its 2-D Fourier transform satisfies equation (18). No other restrictions are applied to D(x,y).

4.4.1 Optical Analysis

A diagram of the proposed system is depicted in figure 10. As before we find its transfer function applying the same method used with the previous systems. All variables are defined as in system 2 except that D(x,y) must be even.

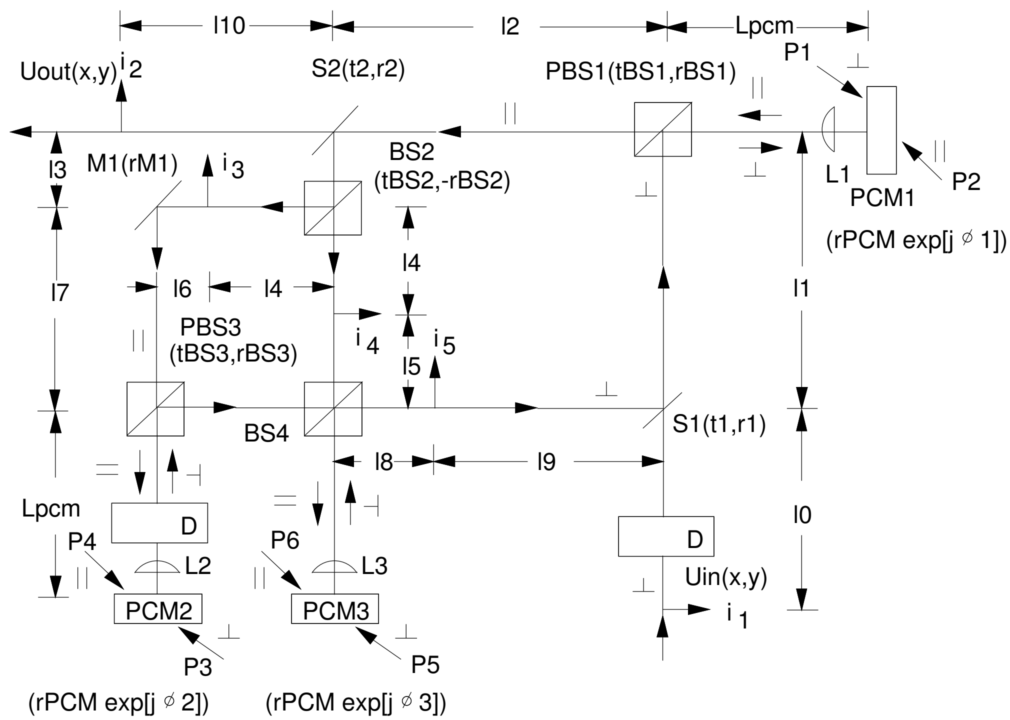

Fig. 10: Optical image restoration system 3

As with system 2, we will assume that the imaging conditions are satisfied to simplify matters. That is, the following conditions are satisfied

$$l_o + l_1 = l_2 + l_{10} \tag{49}$$

$$l_o + l_1 = l_2 + l_3 + l_4 \tag{50}$$

$$l_6 + l_7 = l_6 + l_4 + l_5 \tag{51}$$

$$l_5 = l_8 \tag{52}$$

and

$$l_9 = l_o \tag{53}$$

Under these circumstances we repeat the same procedure used with systems 1 and 2.

First Pass:

$$\begin{aligned} U_{\text{out}}(f_x,f_y) &= t_1 t_2 r_{\text{BS1}} t_{\text{BS1}} r_{\text{PCM1}} e^{j\left(\phi_1 - \phi_o\right)} D^*(f_x,f_y) U_{\text{o}}(f_x,f_y) \\ &= U_1(f_x,f_y) \end{aligned}$$

Second Pass:

$$\begin{aligned} U_{\text{out}}(f_x,f_y) &= U_1(f_x,f_y) + t_1 t_2 r_{\text{BS1}}^2 t_{\text{BS1}}^2 r_{\text{PCM1}}^2 r_1 r_2 t_{\text{BS4}} \\ &\quad \left\{ t_{\text{BS2}} r_{\text{BS4}} r_{\text{PCM3}} e^{-j\phi_3} - r_{\text{BS2}} r_{\text{M1}} t_{\text{BS3}} r_{\text{BS3}} r_{\text{PCM2}} e^{-j\phi_2} | D(f_x,f_y) |^2 \right\} \\ &\quad U_{\text{o}}(f_x,f_y) D^*(f_x,f_y) e^{j\left(2\phi_1 - \phi_o\right)} \\ &= U_1(f_x,f_y) + U_2(f_x,f_y) \end{aligned}$$

Third Pass:

$$\begin{aligned} U_{\text{out}}(f_x,f_y) &= U_1(f_x,f_y) + U_2(f_x,f_y) + t_1 t_2 r_{\text{BS1}}^3 t_{\text{BS1}}^3 r_{\text{PCM1}}^3 r_1^2 r_2^2 t_{\text{BS4}}^2 \\ &\quad \left\{ t_{\text{BS2}} r_{\text{BS4}} r_{\text{PCM3}} e^{-j\phi_3} - r_{\text{BS2}} r_{\text{M1}} t_{\text{BS3}} r_{\text{BS3}} r_{\text{PCM2}} e^{-j\phi_2} | D(f_x,f_y) |^2 \right\}^2 \\ &\quad U_{\text{o}}(f_x,f_y) D^*(f_x,f_y) e^{j\left(3\phi_1 - \phi_o\right)} \end{aligned}$$

By setting

$$\beta = \frac{r_{\text{BS2}} r_{\text{M1}} t_{\text{BS3}} r_{\text{BS3}} r_{\text{PCM2}} e^{j\left(\phi_3 - \phi_2\right)}}{t_{\text{BS2}} r_{\text{BS4}} r_{\text{PCM3}}}$$

$$r_{\mathrm{S1}} = t_{\mathrm{BS1}} r_{\mathrm{BS1}} r_1 r_2 t_{\mathrm{BS2}} r_{\mathrm{BS4}} t_{\mathrm{BS4}}$$

$$r_{\mathrm{PCM}} = r_{\mathrm{PCM1}} r_{\mathrm{PCM3}}$$

we obtain at the Nth Pass:

$$\frac{U_{\mathrm{out}}(f_x,f_y)}{U_{\mathrm{o}}(f_x,f_y)} = t_1 t_2 r_{\mathrm{BS1}} t_{\mathrm{BS1}} r_{\mathrm{PCM1}} e^{j\left(\phi_1 - \phi_o\right)} D^*(f_x,f_y)$$

$$\{1 + r_{\mathrm{S1}} r_{\mathrm{PCM}} e^{j\left(\phi_1 - \phi_3\right)} (1 - \beta | D(f_x,f_y) |^2)$$

$$+ r_{\mathrm{S1}}^2 r_{\mathrm{PCM}}^2 e^{j2\left(\phi_1 - \phi_3\right)} (1 - \beta | D(f_x,f_y) |^2)^2 + \ldots.\}$$

If the condition

$$\left| r_{\mathrm{S1}} r_{\mathrm{PCM}} (1 - \beta | D(f_x,f_y) |^2) e^{j\left(\phi_1 - \phi_3\right)} \right| < 1 \tag{54}$$

is satisfied, we obtain

$$\frac{U_{\mathrm{out}}(f_x,f_y)}{U_{\mathrm{o}}(f_x,f_y)} = \frac{t_1 t_2 r_{\mathrm{BS1}} t_{\mathrm{BS1}} r_{\mathrm{PCM1}} e^{j\left(\phi_1 - \phi_o\right)} D^*(f_x,f_y)}{1 - r_{\mathrm{S1}} r_{\mathrm{PCM}} e^{j(\phi_1 - \phi_3)} (1 - \beta | D(f_x,f_y) |^2)}$$

So the intensity transfer function is given by

$$T(f_x,f_y) = \left| \frac{U_{\mathrm{out}}(f_x,f_y)}{U_{\mathrm{o}}(f_x,f_y)} \right|^2$$

$$= (t_1 t_2 r_{\mathrm{BS1}} t_{\mathrm{BS1}} r_{\mathrm{PCM1}})^2 \left| \frac{e^{j\left(\phi_1 - \phi_o\right)} D^*(f_x,f_y)}{1 - r_{\mathrm{S1}} r_{\mathrm{PCM}} e^{j(\phi_1 - \phi_3)} (1 - \beta | D(f_x,f_y) |^2)} \right|^2$$

To get the desired result it is required that

$$r_{\mathrm{S1}} r_{\mathrm{PCM}} = 1 \quad \text{and} \quad \phi_3 = \phi_1 \tag{55}$$

Then the intensity transfer function is equal to

$$T(f_x,f_y) = (t_1 t_2 r_{\mathrm{BS1}} t_{\mathrm{BS1}} r_{\mathrm{PCM1}})^2 \left| \frac{e^{j\left(\phi_1 - \phi_o\right)} D^*(f_x,f_y)}{\beta \left| D(f_x,f_y) \right|^2} \right|^2$$

or

$$T(f_x,f_y) = \left| \frac{t_1 t_2}{r_1 r_2 r_{\mathrm{BS2}} t_{\mathrm{BS2}} r_{\mathrm{BS3}} t_{\mathrm{BS3}} r_{\mathrm{M1}} r_{\mathrm{PCM2}} \left| D(f_x,f_y) \right|} \right|^2 \qquad (56)$$

Once more, provided that conditions (49), (50), (51) (52) (53) and (55) are met and that the convergence condition expressed by (54) is satisfied, our system has a transfer function which is the inverse of $D(f_x,f_y)$, multiplied by a constant gain factor.

4.4.2 Geometry Analysis

As we have done before, we analyze now the system geometry to ensure that it meets the geometry requirements which are given by (49), (50), (51) (52) and (53). Again it is easy to see the relationship between the sides of the system, by referring to figure 11.

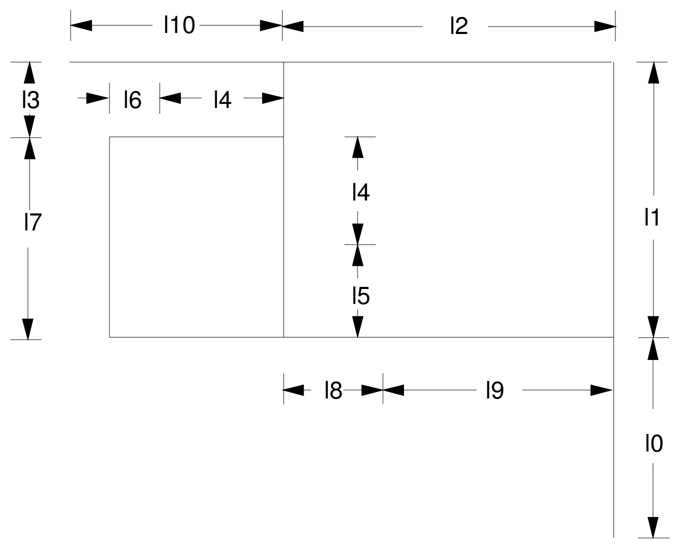

Fig. 11: System 3 geometry

According to this figure we can write:

$$l_1 = l_3 + l_4 + l_5 \qquad (57)$$

$$l_2 = l_8 + l_9 \tag{58}$$

$$l_7 = l_4 + l_5 \tag{59}$$

By combining (52) with (59) we get

$$l_7 = l_4 + l_8$$

which agrees with the imaging condition expressed by equation (51).

Now, applying (57) (58) and (52) to (50) we obtain

$$l_o + l_3 + l_4 + l_5 = l_9 + l_5 + l_3 + l_4$$

which reduces to equation (53). It follows that the system's geometry is consistent with the required imaging conditions. To find the correct distances to the input and output planes, l_o and l_{10} given respectively by equations (50) and (49), can be used.

5 Conclusion

Different schemes for the optical implementation of iterative algorithms for image restoration have been discussed. Experimental results and conditions for the operation of such experiments, as well as known difficulties have been reported.

First a hybrid TV-optical approach was introduced, which allows the restoration of images by implementing the Jacobi method using a TV-optical feedback scheme. Experiments performed with this type of system using an electronically simulated distortion, demonstrated a noticeable improvement in image quality, with convergence times in the order of one second. However, for non-simulated distortions, major difficulties arise from the fact that two of the cameras utilized must be perfectly aligned, identically matched and synchronized with each other as well as with the monitor. Also, the utilization of this system is limited by the convergence conditions of the Jacobi method, that is, it can only undo distortions with positive transfer functions.

Next, a modification to the TV-optical approach was introduced which utilizes only one camera and a minicomputer to process the information received from it. This optical-digital scheme overcomes the limitations of the first system, that is, the need for synchronization and perfect alignment between camera and monitor, by digitally averaging the images received from the camera. Also, this scheme allows the use of the Jacobi algorithm with reblurring in addition to the regular Jacobi algorithm and therefore overcomes the positivity limitation. However, this system requires a long time to converge (70 to 150 sec. per iteration) as compared to the TV-optical system, since it has to digitally average many frames in order to acquire an image.

To overcome the difficulties encountered with the hybrid schemes, a new totally optical implementation has been proposed. By utilizing a Phase conjugate mirror (PCM) as an amplifying element, a coherent optical feedback system can be used to perform iterative image restoration algorithms. The convergence time of such a system is limited only by the response time of the PCM.

Three different systems have been proposed and analyzed. Each system is more complex than its predecessor, but imposes less restrictions on the type of distortion that can be undone by employing it. Therefore, each one has its applications. For distortions with real spatial impulse response, system 1 offers the simplest setup to recover the original image. On the other hand, if the distortion has a complex impulse response, then system 2 can provide the desired results. Finally, if the spatial transfer function of the distortion has negative values, then system 3 can undo it’s effects by implementing the Jacobi method with reblurring.

It has been left to future researchers to actually build and experiment with the proposed totally optical systems. Foreseeable difficulties include achieving the required phase relationships between the beams and overcoming possible inaccuracies in the geometry of the system setups. Also for the future, other modifications may be introduced to allow, for example, the implementation of image restoration algorithms that incorporate prior knowledge about the solution in the form of constraints. Such algorithms are discussed in detail in [2].

6 References

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